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GEOMETRICALLY NON-LINEAR PROBLEMS OF THE BENDING OF ORTHOTROPIC ANNULAR AND SECTORAL PLATES†

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Research on calculating the geometrically non-linear stress–strain state of thin elastic orthotropic annular and sectoral plates, acted upon by static loads, is reviewed. The research considered is classified with respect to the form of the loading and clamping of the plates and with respect to their geometrical characteristics. It is shown that this area of the theory of plates, despite the fact that it appears to have been fully investigated, cannot be regarded as completed.

Phenomena connected with the geometrical non-linearity of the behaviour of circular plates were encountered for the first time by Wilson [1] in 1877, when he carried out tests on the flat bottoms of steel cylindrical steam boilers. However, the first theoretical investigations in this area considered the strength of plates, taking into account the geometrical non-linearity – the finiteness of their deflections only saw the light in 1902. These were the papers by Bubnov [2–5] on the steel hull of ships, in which he used, for the first time, Kirchhoff's quadratic deformation relations [6] to obtain the equations of equilibrium of a plate. Bubnov's equations are identical with Berger's equations [7], apart from certain constants, and anticipate Föppl's equations [8] and Karman's equations [9–10].

More than one and a half thousand publications are known which investigate the bending, stability and vibrations of elastic circular and annular plates of different internal structure, taking into account the geometrical non-linearity of their behaviour. One of the particular areas of this mass of research in the non-linear theory of elastic plates is a group of papers which calculate the bending of elastic orthotropic circular and annular plates. The foundations of the linear theory of the bending of thin orthotropic and anisotropic plates were laid by Gehring [11], Boussinesq [12] and Huber [13, 14] in the first half of the nineteenth century and beginning of the twentieth century. However, the equations of the bending of orthotropic plates of finite deflections, based on Kirchhoff's deformation relations

$$\frac{1}{E_2}\frac{\partial^4 F}{\partial x^4}+\left(\frac{1}{G}-\frac{2\nu_1}{E_1}\right)\frac{\partial^4 F}{\partial x^2\partial y^2}+\frac{1}{E_1}\frac{\partial^4 F}{\partial y^4} = \left(\frac{\partial^2 w}{\partial x\partial y}\right)^2-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}$$

$$D_1\frac{\partial^4 w}{\partial x^4}+2D_3\frac{\partial^4 w}{\partial x^2\partial y^2}+D_2\frac{\partial^4 w}{\partial y^4} = q+h\left(\frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2}-2\frac{\partial^2 F}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}+\frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right)$$

$$(0\le x\le a,\ 0\le y\le b)$$

where F and w are functions of the stresses and deflections of the plate, q is the distribution of the transverse load, E_1, E_2, ν_1, ν_2 and G are Young's moduli, Poisson's ratio and the shear modulus for the principal directions of elasticity of the plate material, $D_1 = E_1h^3/[12(1-\nu_1\nu_2)]$, $D_2 = E_2h^3/[12(1-\nu_1\nu_2)]$ and $D_k = Gh^3/12$ are the principal bending and twisting stiffnesses, a, b and h are the length, width and thickness of the plate, and $D_3 = D_1\nu_2 + 2D_k$, were only obtained by Rostovtsev in 1940. These equations reduce to the Föppl–Karman equations in the case of isotropic plates $E_1 = E_2 = E$, $\nu_1 = \nu_2 = \nu$, $G = E/2(1+\nu)$.

Although we have attributed the equations of the bending of orthotropic plates of finite deflections to Rostovtsev, we must recall the research carried out by Mushtari. In 1938, in his dissertation [16], he obtained quadratic deformation relations and the specific forces and moments, and derived the equations

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of equilibrium in forces for a thin orthotropic shell. These equations, when reduced to the case of a plate and written in mixed form, are identical with Rostovtsev's equations. However this was not done in his dissertation.

The common view on the theory of orthotropic and anisotropic plates, including geometrically non-linear problems, and on the present position in this area of mechanics, can be found in the books [17–23] and reviews [24–26]. The present review is devoted to one of the comparatively small areas of the non-linear theory of plates – the bending of orthotropic annular and sectoral plates. It is an extension of the review given in [27] and covers the greatest number of papers on this problem compared with the references mentioned above. The stability and vibrations of geometrically non-linear orthotropic plates is not touched on in this review, but we do take into account all the existing papers on these topics, in which the bending of plates is a special case of the researches carried out.

1. ANNULAR PLATES

A uniformly distributed transverse load

The solution of the problem of the axisymmetric deformation of an annular cylindrical orthotropic plate of constant thickness, acted upon by a uniform transverse pressure, was obtained in [28–31]. All the researchers used Rostovtsev's equations, written in a cylindrical system of coordinates. Andreyeva [28], in considering a corrugated plate, first reduced it to a theoretical cylindrical orthotropic scheme; the plate was assumed to be rigidly clamped along the external contour and had an absolutely rigid central core, the presence of which corresponds to a rigid clamp movable in the transverse direction. Other researchers considered the case when the plate was either rigidly clamped or subject to sliding clamping along both contours [29], and also the case of a plate with free or external couplings along the internal contour and rigidly clamped or supported on a hinge along the external contour [30]. To solve this problem the method used [28] was to separate the solution [32] into a bending (linear) and a membrane (non-linear) solution [32]. An exact solution was written for the bending component and the Bubnov–Papkovich solution for the membrane solution, representing the deflection by a second-order polynomial. As a result a cubic relation was obtained between the value of the transverse pressure and the deflection along the internal contour of the plate. This solution was subsequently given in the book [33].

Trombski [29] also used the Bubnov–Papkovich method, but entirely for the Rostovtsev equations, written in a mixed form in a cylindrical system of coordinates. The deflection was approximated by a single term in the form of a third-order polynomial, and relations were obtained which were similar to those obtained previously in [28].

The problem was also solved numerically in [30] by a method which is a step-by-step algorithm, set up for linearized equations. The linearized equations were solved by expanding the deflection and the stress function in series in Chebyshev polynomials, and the initial approximation for a new value of the load was calculated using extrapolation with respect to three previous points. Graphs of the deflections and specific radial and circumferential forces and moments at characteristic points as a function of the transverse pressure were given in [30].

A similar algorithm was used in [31] to solve the axisymmetric problem of finite deflections of a cylindrical orthotropic annular plate, loaded by a uniform transverse pressure. The main difference between this algorithm and that employed previously in [30] is the use of the collocation method when solving the linearized Rostovtsev equations with a polynomial representation of the deflection of the plate and its stress function. Graphs were drawn from the results of the calculations in [31] of the relation between the deflections and the radial and circumferential membrane and bending stresses as a function of the applied load for different forms of the boundary conditions and different relative dimensions of the internal opening and the ratio of the moduli of elasticity in the circumferential and radial directions. The relations mentioned above were shown for a free internal contour and a hinge-supported or rigidly clamped external contour, a free internal contour and a rigidly clamped external contour with elasticity in the transverse direction, a free internal contour and a rigidly clamped external contour with specified radial displacements, for a free internal contour and a hinged support external contour with elasticity with respect to the angle of rotation of the normal, for a hinge-supported and clamped plate with an absolutely rigid central support, and for a plate that is free along the external contour and elastic with respect to the angle of rotation of the normal of the support along the internal contour.

A cylindrically orthotropic plate of linearly varying thickness, acted upon by a uniform transverse load was considered in [29]. Using the Bubnov–Papkovich method with a single-term approximation of the deflections by a polynomial, an axisymmetric solution of Rostovtsev's equations was constructed for the case of simultaneous either rigid or sliding clamping of both contours of the plate, and an analytical relation was obtained between the deflections of the plate and the load applied to it. In the same paper, a similar solution was derived for a cylindrically orthotropic annular plate of constant thickness, with rigid or sliding clamping of both contours, having an initial irregularity similar to deflection, specified using an approximation.

A solution of the problem of finite deflections of a cylindrically orthotropic plate lying on an elastic foundation and acted upon by a uniform transverse load was constructed numerically in [34]. The plate has a free internal contour and a rigidly clamped or a hinge-supported external contour, and a model having three terms, one of which is proportional to the first power of the deflection, the second of which is proportional to the cube of the deflection, while the third is proportional to the Laplace operator of the deflection, is used as the mathematical model of the elastic foundation. The algorithm for solving this boundary-value problem consists in setting up a process of successive loadings in the form of extrapolation of the initial approximation at the three previous points, used together with an iterational refining process, the basis of which is the solution of the linearized boundary-value problem for Rostovtsev's equations. The collocation method with an approximation of the deflection and of the stress function by polynomials was used to solve the linearized equations, in which the zeros of the Legendre polynomials were chosen as the collocation points. Graphs of the distribution of the deflections, the radial and circumferential stresses along the radius and the dependence of the greatest deflection of the plate on the value of the transverse pressure were constructed.

A non-uniformly distributed transverse load

The non-symmetrical behaviour of a cylindrically orthotropic annual plate when acted upon by a transverse load, which is constant along the radius and varies cosinusoidally in the circumferential direction, was investigated in [35] using Rostovtsev's equations. To solve the problem for rigid clamping of the external contour of the plate and an internal contour free from external couplings, the method of straight lines was used along the circumferential coordinate together with the method of orthogonal pivotal condensation along the radius, which employs linearization of the equations of Newton's method. As an illustration of the calculations, graphs were presented of the distribution of the radial bending moment and the deflection along the radius of the plate for a zero value of the circumferential coordinate and different values of the ratio of the moduli of elasticity in the radial and circumferential directions.

A uniformly distributed transverse load on the internal contour (a concentrated force on a rigid core)

The case of axisymmetric deformation of an annular cylindrically orthotropic plate with a transverse load uniformly distributed along the internal contour was analysed using Rostovtsev's equations in [28, 36, 37]. The cases when a distributed transverse load was applied to the internal and external contours of the plate were considered in the previously mentioned paper [31].

A corrugated plate, reduced to a cylindrically orthotropic plate, rigidly clamped along the external contour and having an absolutely rigid central core with a concentrated force applied to it, was considered in [28]. The presence of a core with a force corresponds to rigid clamping of the internal contour, mobile in the transverse direction, with a transverse load distributed along it. To solve the problem, the solution was separated [32] into a bending term, described by linear equations for an orthotropic plate, and a membrane term, which is obtained from the solution of Föppl's equations for an orthotropic absolutely flexible membrane. An accurate analytical solution is obtained for the bending component, and a solution is constructed for the membrane component by the Bubnov–Papkovich method with the deflections represented by a linear function. As a result a relation was obtained between the concentrated force and the deflections on the internal contour of the plate.

A similar problem was solved by Dolgopolov in [36]. It differs from the problem considered by Andreyeva [28], in that a cylindrically orthotropic plate was initially considered and the thickness of the plate was assumed to be variable, the variation being described by a power law. The solution of this problem is constructed by two methods: the Bubnov–Papkovich method, with a single-term approximation of the deflections by a linear solution, and the small-parameter method, by representing the deflections and the radial forces in the plate in the form of series in powers of the concentrated force. Relations similar to those obtained by Andreyeva were obtained [28].

An annular cylindrically orthotropic plate of constant thickness with an absolutely rigid central core, at the centre of which a concentrated force is applied, was also considered in [37]. The same formulation of the problem was used, but the initial axisymmetric deflections of the plate, which were described by a third-order polynomial, were taken into account, and all the classical boundary conditions on the external contour of the plate were analysed. A solution of the problem was obtained by the Bubnov–Papkovich method with a single-term approximation of the deflections similar to the representation of the initial deflections, and a graph of the deflections on the internal contour of the plate as a function of the applied force was constructed.

Dumir *et al.* [31], whose solution is described above, constructed graphs of the deflections and radial and circumferential membrane and bending stresses as a function of the applied load for hinge-supported and rigidly clamped plates along the external contour and loading with a transverse load and a freely displaceable internal contour. This was done for a plate, elastically supported with respect to the angle of rotation of the normal on the internal contour and loaded by a transverse load on the free external contour, and for a hinge-supported and a rigidly clamped plate with an absolutely rigid core with a transverse concentrated force.

A turning out (warping) reversing moment on the central insert

The problem of deforming an annular plate out of the plane in which it is rigidly clamped or subject to sliding clamping along the external contour by a moment whose vector lies in the plane of the plate and which is applied to an absolutely rigid central insert, rigidly or slidingly connected to the plate along its internal contour respectively, was solved in [38]. Rostovtsev's equations in an asymmetrical formulation in a cylindrical system of coordinates were used to describe the problem for a plate of constant thickness. The solution of these equations was constructed by the Bubnov–Papkovich method, for which the deflections of the plate are represented by the product of a third-order polynomial of the actual radius of the plate and the cosine of the circumferential coordinate. The results of the calculation are represented by a graph of the change in the moment applied to the core against its angle of rotation from the plane of the plate.

The heating of a plate

Rostovtsev's equations in mixed form were used in [39, 40] to describe the axisymmetrical deformation of an annular finely perforated plate heated non-uniformly over the thickness and along the radius. The initial axisymmetric imperfection of the plate and the variability of the coefficient of linear expansion and the moduli of elasticity of the plate material in the radial and circumferential directions are taken into account. A solution is obtained by the method of successive approximations for a plate which is elastically supported with respect to radial displacements and with respect to the angle of rotation of the normal along both contours and rigidly supported with respect to deflection on one of them and elastically on the other. There are no calculations in these publications.

Circumferential radial forces and the initial imperfections. The problem of the finite deflections of annular cylindrical orthotropic plates with an initial axisymmetric imperfections, which manifest themselves when acted upon by a uniform radial load, applied to one or two contours of the plate, was considered in an axisymmetrical formulation using Rostovtsev's equations in [29, 41].

This problem was solved for a densely perforated plate by reducing it to the cylindrically orthotropic case with sliding clamping along the external contour, when a radial load is applied to it and when the internal contour is free from external constraints [41]. A plate loaded in a radial direction and having sliding clamping on both contours was also considered in [29]. In both papers the Bubnov-Papkovich method with a single-term representation of the deflections and of the initial imperfections of the plate by polynomials was used. Graphs of the deflection as a function of the value of the load and of the amplitude of the initial imperfection were drawn from the results.

Combinations of loads

The axisymmetric deformation of a cylindrically orthotropic annular plate under the combined action of a uniform transverse pressure and radial forces, uniformly distribution over the external contour, were considered in [42]. The behaviour of the plate with a free internal contour and a free support or sliding clamping or the external contour was described using Rostovtsev's equations, for which the boundary-value problem was solved using a step-by-step algorithm, based on linearization of the resolvents. The linearized equations themselves were solved using an expansion of the deflection and of the stress function in series in Chebyshev polynomials, while the initial approximation of the solution with a new value of the load was extrapolated with respect to three previous points. From the results of the calculations graphs were drawn showing how the deflection depends on the value of the compressing force and how the radial and circumferential membrane and bending stresses depend on the value of the deflection along the internal contour of the plate.

We can take the investigation carried out by Liu Ren-huai [43, 44] as an example of research in which the axisymmetric behaviour of annular cylindrically orthotropic plates of constant thickness were considered when they are simultaneously acted upon by transverse contour forces, a bending contour moment and circumferential radial forces on the internal contour, and also transverse contour forces, and a bending contour moment, as well as circumferential radial forces and a distributed transverse load. In this research, to solve the problem of the deformation of a composed rigidly clamped annular corrugated plate, acted upon by a central concentrated force or a concentrated force and a uniform transverse pressure, it is necessary to use the small-parameter method to construct a solution for an annular cylindrically orthotropic plate with rigid clamping along the external contour and arbitrary values of the force factors on the internal contour. This solution can be used when carrying out calculations for the cases of the loading of annular plates considered.

2. SECTORAL PLATES

The problem of the finite deflections of a cylindrically orthotropic plate, constructed in the form of an annular sector was solved in [45]. The plate was assumed to be rigidly clamped over the whole contour and was loaded by a uniform transverse pressure. The stress–strain state was described by Rostovtsev's equations in displacements. The following version of the method of finite differences was employed to divide the problem into discrete parts. A polar-orthogonal grid was superimposed on the plane region occupied by the plate, and the derivatives of the displacements along the circumferential coordinate $\partial^2 u/\partial\theta^2$, $\partial^2 \upsilon/\partial\theta^2$ and $\partial^4 w/\partial\theta^4$ were taken as the unknown solutions at the nodes of this grid. Using Green's function for the linear problem in fractions of the values of these derivatives at the nodes of the grid, the displacements themselves and the remaining derivatives were found, and then all the information on the displacements were substituted into the resolvents, also written for each of the nodal points of the grid. As a result a system of non-linear algebraic equations in the chosen unknowns is obtained, which is solved by the method of successive loadings and the solution is refined for each value of the load by Newton's method. As an illustration of these calculations, graphs were presented of the deflections of a plate and its radial and circumferential membrane and bending stresses as a function of the transverse load for different values of the polar aperture angle of the plate and the ratio of the radii of its larger and smaller bases, and also in the form of tables which showed the convergence of the calculations of the deflections and the specific forces and moments as a function of the density of the grid employed.

3. CONCLUSION

The number of papers on the geometrically non-linear behaviour of annular orthotropic plates when acted upon by static loads is much less than for circular plates (see [27]). Russian scientists also have priority in this area, their papers were published at the end of the 1950s, whereas the main body of work on this topic belongs to the 1970s and 1980s.

Despite the limited number of papers on annular plates, practically all the classical cases of their loading and clamping are analysed in them, as well as the cases of thickness variability, and the case when there is initial imperfection in them. The only exceptions are certain cases of the application of combined loads containing shear loads in the plane of the plate. From this point of view it can be asserted that these problems have been transferred from the sphere of being of solely scientific interest to the sphere of interest of the practical design of engineering structures, if such emerge.

However, bearing in mind the lack of solutions of problems for annular plates with large deflections for small and finite deformations, investigations which take into account the simplifying Berger hypotheses, and also investigations of the behaviour of circular plates with non-cylindrical (rectangular) orthotropic material, one cannot regard this area of the theory of plates as being complete, and the need for further research in this area is quite obvious.

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